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Solids concentration profiles and pressure drop in pipeline flow of multisized particulate slurries

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Abstract

Concentration profiles for six particle sizes ranging from 38 to 739 μm were measured using a traversing mechanism and isokinetic sampling probe at nine levels in the vertical plane for multisized particulate zinc tailings slurry flowing through 105 mm diameter horizontal pipe. Experiments were conducted at three flow velocities of 2, 2.75 and 3.5 m/s using five efflux concentrations ranging from 4% to 26% by volume for each velocity. Solids concentration profiles were found to be a function of particle size, velocity of flow and efflux concentration of slurry. Solids concentration varied with the vertical position, except for particle size of 38 μm. Experimental data for pressure drop were also collected at five efflux concentrations ranging from 4% to 26% at flow velocities ranging from 1.2 to 4.0 m/s for each efflux concentration. Karabelas model for solids concentration profiles and Wasp model for pressure drop have been modified by alleviating some of the restrictive assumptions used in the models. The modified models are compared with the experimental data collected in the present study and show good agreements. Comparison of pressure drop data with the predictions by two layer model is also satisfactory.

Keywords: Pressure drop; Concentration profile; Particle diffusivity; Multisized particulate slurry; Efflux concentration; Particle size distribution; Static settled concentration

1. Introduction

The transportation of slurries through pipes are normally used for long distance. The design procedure for the pipeline transporting multisized slurries is very complex. Some of the most

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important design parameters for slurry pipelines are solids concentration profiles, pressure drop and deposition velocity. Researchers over the years have been able to predict these parameters based on the extensive experimental data and use them as design tools.

O'Brien (1933) and Rouse (1937) were the first ones to propose the simple diffusion type model for the prediction of concentration profile of solids in turbulent streams. Ismail (1952) modified the simple diffusion model by correlating the mass transfer coefficient to shear stress/velocity gradient which in turn was deduced from von Karman universal velocity profile and then compared the predictions with his own experimental results generated in a rectangular channel. Ismail (1952) found that the von Karman constant κ was a function of solid concentration and decreased with increasing solid concentration. Wasp (1963) assuming a value of κ to be 0.35 in Ismail's equation found that the prediction of concentration profile agreed reasonably well with his experimental results. Subsequently Wasp et al. (1970) analysed their own data and Ismail (1952) data at $y = 0.5D$ and $0.08D$, where y is the distance from the pipe bottom and D is the pipe diameter, to come up with a modified equation for the prediction of concentration profile. Parallel to Wasp work, Shook and Daniel (1965) and Shook et al. (1968) were involved in extensive research in the area of slurry flow in pipes. They found that the equation proposed by O'Brien (1933) and Rouse (1937) does not account for normal dispersive forces. Karabelas (1977) in the late seventies developed an empirical model to predict the vertical composite concentration and particle distributions based on Hunt's (1954) formulation for solid liquid flows through pipe. Karabelas (1977) has also shown the applicability of his closed form expression by comparing the predictions with experimental results. Seshadri et al. (1982) used the equation developed by Wasp et al. (1970) to predict the overall concentration profile and individual size distribution and found the agreement to be satisfactory in the top of the pipe whereas large deviations were observed at the bottom of the pipe. Efforts are still on to develop a reasonable model for the prediction of concentration profile in pipes and in this direction, the work of Roco and Shook (1983, 1984), Gillies et al. (1999, 2000) and Kaushal et al. (2002) is worth mentioning. Most of the equations available in literature for predicting vertical solids concentration profiles in slurry pipeline are empirical in nature and have been developed based on limited data on materials having equisized or narrow size-range particles. These conditions are rarely encountered in commercial slurries. The particle size distribution in commercial slurries is invariably very wide encompassing three orders of magnitude. From the literature it is evident that the flow for commercial slurry pipelines is turbulent with heterogeneous distribution of solid particles across the pipe cross-section. In the light of the above shortcomings, an attempt has been made in the present study to modify Karabelas (1977) model for solids concentration profiles by alleviating some of the restrictive assumptions used in the model.

Several studies for pressure drop prediction are available in literature (Wasp et al., 1977; Doron et al., 1987; Gillies et al., 1991; Sundqvist et al., 1996; Mishra et al., 1998; Ghanta and Purohit, 1999; Wilson et al., 2002, etc.). It has been found that the two phase (vehicle-bed) approach proposed by Wasp et al. (1977) and two layer model proposed by Gillies et al. (1991) hold out great promise for the prediction of pressure drop in slurry containing multisized particles. An attempt has been made in the present study to modify Wasp et al. (1977) model for pressure drop by alleviating some of the restrictive assumptions used in the model. The predicted pressure drops by two layer model proposed by Gillies et al. (1991) have also been compared with the experimental data collected in the present study.

2. Brief description of existing models

2.1. Karabelas (1977) model for solids concentration profiles

Karabelas (1977) proposed the following equation for solids concentration profiles for the flow of multisized particulate slurries through horizontal pipe:

$$C_j(y) = \frac{G_j \exp(-w_j f(y))}{1 + \sum_{i=1}^{n} G_i \exp(-w_i f(y))}, \quad j = 1, 2, \ldots, n \tag{1}$$

where $f(y) = \int \frac{\mathrm{d}y}{\varepsilon_s(y)}$, ε_s is the particle diffusivity, G_j is a set of coefficients characteristic of each size but independent of space coordinates and w_j is the settling velocity of jth size particle. In order to proceed in the development of the distribution function $C_j(y)$ the following two assumptions are made:

(i) The dimensionless eddy diffusivity ξ is a constant independent of solid concentration and space coordinates, that is

$$\varepsilon_s = \xi R u* \tag{2}$$

where R is the radius of pipe, $u*$ is the shear velocity and ε_s is the particle diffusivity.

(ii) The solids concentration $C_j(y)$ is a function of vertical co-ordinate y only.

Karabelas (1977) presented the following final solution for the solids concentration profiles for different sizes :

$$C_j(y^*) = \left[\frac{\bar{v}_j}{E(k_j)} \exp(-k_j y^*)\right]\left[1 + \sum_{i=1}^{n} \frac{\bar{v}_j}{E(k_j)} \exp(-k_i y^*)\right]^{-1} \quad j = 1, 2, \ldots, n \tag{3}$$

where $y^* = \frac{y}{R}$, varies from -1 to $+1$,

$$k_j = \frac{w_{j0}}{\xi u*}; \quad j = 1, 2, \ldots, n \tag{4}$$

where w_{j0} is the unhindered settling velocity of of jth size particle,

$$E(k_j) = \frac{1}{A}\int_{\mathrm{A}} \exp(-k_j y^*)\,\mathrm{d}A; \quad j = 1, 2, \ldots, n \tag{5}$$

$$\bar{v}_j = \frac{C_{\mathrm{v}j\mathrm{f}}}{1 - \sum_i C_{\mathrm{v}i\mathrm{f}}} = \frac{C_{\mathrm{v}j\mathrm{f}}}{1 - C_{\mathrm{vf}}}; \quad j = 1, 2, \ldots, \ldots, n \tag{6}$$

where C_{vf} is the efflux or overall average concentration by volume, $C_{\mathrm{v}j\mathrm{f}}$ is the average concentration by volume of jth size particle and $C_j(y)$ is the local concentration of jth size particle.

Computer program for the Karabelas (1977) final solution given by Eq. (3) was developed. The solid particles were divided into six sizes and the unhindered settling velocity w_{jo} for the jth size

Table 1
Drag relationships

Fall regime and range of particle Reynolds number (Re_d)	Relation for drag coefficient (C_D)
Stoke's law	
$Re_d \leqslant 1.0$	$C_D = 24Re_d^{-1}$
Intermediate	
$1 < Re_d \leqslant 1000$	$C_D = 24Re_d^{-1}(1 + 0.15Re_d^{0.687})$
Newton's law	
$1000 < Re_d \leqslant 2 \times 10^5$	$C_D = 0.44$

particle was calculated using the drag relationships given in Table 1, where Re_d is the particle Reynolds number.

The value of shear velocity $u*$ required in Karabelas model is evaluated as

$$u* = \sqrt{gri_m} \tag{7}$$

where r is the hydraulic radius of pipe and i_m is the pressure drop in terms of m of water column per unit length of pipe measured at the given concentration and flow velocity.

2.2. *Wasp et al. (1977) model for pressure drop*

The method is based on the assumption that the total pressure drop in two phase flow can be split into two parts; pressure drop due to vehicle (homogeneously distributed particles) and excess pressure drop due to bed formation (heterogeneously distributed particles). The method suggested is an iterative one. In the first iteration, the particular suspension is assumed to be completely homogeneous. Based on this assumption the pressure gradient is computed using Darcy–Weisbach formula which for slurry flow is given by:

$$i_{\text{vehicle}} = \frac{2f_m V_m^2}{gD} \tag{8}$$

where i_{vehicle} is the pressure gradient due to homogeneously distributed particles (vehicle) and f_m is the Fanning friction factor has been evaluated by Wood's equation:

$$f_m = a + bRe_m^{-c} \tag{9}$$

where $a = 0.026(\varepsilon/D)^{0.225} + 0.133(\varepsilon/D)$, $b = 22(\varepsilon/D)^{0.44}$, $c = 1.62(\varepsilon/D)^{0.134}$, (ε/D) is the relative pipe roughness is determined from the velocity vs. pressure drop data for water, generally taken before start of each experimental run with slurry and Re_m is the Reynolds number for the slurry is calculated using Thomas (1965) correlation for slurry viscosity μ_m as given below:

$$Re_m = \frac{\rho_m V_m D}{\mu_m} \tag{10}$$

$$\frac{\mu_m}{\mu_l} = 1 + 2.5C_{\text{vf}} + 10.05C_{\text{vf}}^2 + 0.00273\exp(16.6C_{\text{vf}}) \tag{11}$$

where ρ_m is the mass density of slurry, V_m is the flow velocity, g is the acceleration due to gravity, μ_l is the viscosity of carrier liquid and C_{vf} is the slurry efflux concentration by volume.

In the second iteration particles in slurry are divided into 4–6 sizes. For each size fraction the percentage of solids in vehicle is calculated as

$$\log\frac{C}{C_A} = -1.8\frac{w_j}{\beta\kappa u*} \tag{12}$$

where C/C_A is the ratio of volumetric concentration of solids at $0.08D$ from top to that at pipe axis, β is the dimensionless particle diffusivity and is taken as 1.0, $\kappa = 0.4$ and is defined as von Karman coefficient, $u*$ is the friction velocity calculated from the pressure drop in first iteration and w_j is settling velocity of jth size particle calculated using standard drag relationships given in Table 1.

After computing the vehicle portion for each particle size, the total percentage of solids in vehicle is calculated. The vehicle pressure drop is calculated for slurry of this concentration using Darcy–Weisbach equation. The pressure drop due to remaining solids in each particle size which are in bed is calculated using Durand type of relationship given as:

$$i_{j\text{bed}} = 82 i_w C_{\text{v}j\text{bed}}\left\{\frac{gD(S-1)}{V_m^2\sqrt{C_D}}\right\}^{1.5} \tag{13}$$

where i_w is the pressure gradient due to flow of carrier liquid, $C_{\text{v}j\text{bed}}$ is the volumetric concentration of bed portion of jth size particle and S is the specific gravity of solids.

The total pressure drop due to bed is the sum of pressure drop due to each particle size in the bed:

$$i_{\text{bed}} = \sum_{j=1}^{n} i_{j\text{bed}} \tag{14}$$

The total pressure drop due to slurry is the sum of pressure drop due to vehicle and total pressure drop due to bed. If the difference in pressure drops in the two successive iterations is greater than ±5%, the iterative scheme is carried to the third iteration. In the third iteration, steps of the second iteration are repeated with modified frictional velocity u_* based on pressure drop in second iteration. Pressure drop obtained from the third iteration is again compared with the pressure drops from the previous iteration and if the difference is still not within acceptable limit (±5%), scheme is again repeated.

2.3. Two layer model proposed by Gillies et al. (1991) for pressure drop

The model consists essentially of mass balances and force balances for the two superimposed slurry layers. The lower layer consists of contact load contributing to sliding friction at the pipe wall and upper layer comprises of suspended load for which the immersed weight is transferred to the carrier fluid. The mass balances relate the mean velocity V and delivered solids volume fraction C_v to the mean velocities V_1 and V_2 and concentrations C_1 and C_{lim} in upper layer and lower layer, respectively

$$AV = A_1V_1 + A_2V_2 \tag{15}$$

$$C_vAV = C_1A_1V_1 + C_{\text{lim}}A_2V_2 \tag{16}$$

where A, A_1 and A_2 are the cross-sectional area of pipe, the area of upper layer and the area of lower layer, respectively.

The concentration C_1 is considered to represent particles which are suspended by fluid lift forces. Concentration C_{lim} is comprised of particles suspended by these lift forces and particles which produce Coulombic friction. The force balances relate the axial pressure gradient to the stresses at the perimeters of the layers. The upper layer is considered to contribute only kinetic friction, so that for horizontal flow:

$$i_m = (\tau_1 S_1 + \tau_{12} S_{12})/(A_1 \rho_1 g) \tag{17}$$

where i_m is the pressure gradient, ρ_1 is the density of liquid, τ_1 is the wall stress of upper layer τ_{12} is the stress at hypothetical interface, S_1 is the partial perimeter of upper layer and S_{12} is the width of interface.

For the lower layer:

$$i_m = (-\tau_{12} S_{12} + \tau_{2k} S_2 + F_{2z})/(A_2 \rho_1 g) \tag{18}$$

where τ_{2k} is the kinetic stress at the partial perimeter of lower layer S_2 and F_{2z} is the Coulombic resisting force exerted by the wall on those particles which do not contribute to the kinetic friction. Assuming the volume fraction of suspended particles is the same in both phases, and that the suspended particles contribute a buoyant effect, the force F_{2z} is given by Wilson (1976) as:

$$F_{2z} = 0.5(\rho_{\mathrm{s}} - \rho_{\mathrm{f}}) C_2 (1 - C_{\mathrm{lim}})(\sin\beta_0 - \beta_0 \cos\beta_0)\eta_{\mathrm{s}}/(1 - C_2) \tag{19}$$

where β_0 is the angle defining lower layer (radians), η_{s} is the coefficient of friction between particle and pipe, ρ_{f} is the density of liquid combined with (−74 μm) fines and ρ_{s} is the density of solids.

The boundary stresses for upper and lower layers were calculated from (density of liquid + suspended particles in each layer) using a Fanning friction factor f_1:

$$\tau_1 = 0.5 f_1 V_1^2 \rho_1 \tag{20}$$

$$\tau_2 S_2 = 0.5 f_1 V_2^2 \rho_{2\mathrm{f}} S_2 \tag{21}$$

where $\rho_{2\mathrm{f}}$ is the density of (fluid + suspended solids) in lower layer.

The interfacial stress τ_{12} is calculated from friction factor f_{12} and difference in velocity between the layers:

$$\tau_{12} = 0.5 f_{12} (V_1 - V_2)^2 \rho_1 \tag{22}$$

The empirical features of the model are the friction factor f_{12}, the friction coefficient η_{s}, the concentration limit C_{lim} and the fraction of the in situ concentration C_{r} which contributes Coulombic friction. f_{12} is calculated using following equation suggested by Wilson (1988):

$$f_{12} = (1 + 2Y)/[(4\log_{10}(D/d_{12}) + 3.36]^2 \tag{23}$$

where $Y = 5 + 1.86\log_{10}(d_{12}/D)$ for (d_{12}/D) is greater than 0.002 and $Y = 0$ otherwise and d_{12} is the particle diameter at the hypothetical interface.

On the basis of experimental measurements, Gillies et al. (1991) determined the value of η_{s} as 0.5 for the sand and gravel slurries. Using Wilson's symbol C_{c} (contact load) for particles, which are not suspended by fluid lift forces, the quantity of interest is

$$(C_{\mathrm{c}}/C_{\mathrm{r}}) = A_2 (C_{\mathrm{lim}} - C_1)/(A C_{\mathrm{r}}) \tag{24}$$

On the basis of experimental data base, C_c/C_r was found to be a function of the ratio of the slurry bulk velocity to particle terminal falling velocity at infinite dilution:

$$(C_c/C_r) = \exp[-0.0184V/v_\infty] \tag{25}$$

where v_∞ is the unhindered settling velocity of particle of diameter d_{50} and d_{50} is the mass median particle diameter of (+74 μm) solids.

For the calculation of C_{lim}, following correlation is used:

$$(C_{max} - C_{lim})/(C_{max} - C_r) = 0.074(V/v_\infty)^{0.44}(1 - C_r)^{0.189} \tag{26}$$

On the basis of experiments, Gillies et al. (1991) suggested the value of C_{max} for narrow and broad distributions as 0.6 and 0.75, respectively.

3. Experimental program

The experiments were conducted using closed-loop pipeline described elsewhere (Kaushal et al., 2002). The pipeline was fabricated from mild steel, with internal diameter 105 mm.

Composite concentration profiles were measured using a sampling tube having a 4×6 mm^2 rectangular slot, 3 mm above the end to collect representative samples at nine levels in the pipe line. Samples are collected from different heights from bottom of the pipe in the vertical plane of the cross-section to determine the concentration profile under near isokinetic conditions. During the collection of concentration samples at various locations it was ensured that the flow of the slurry through the sampling tube outlet is continuous and uniform. If the tube got choked high pressure water was used to open it. Further sufficient time was allowed before sample collection in order to ensure steady state conditions. The sampling tubes are mounted on vernier type of traversing mechanism to enable traversing of the tube from the top to the bottom in vertical plane and its location can be accurately measured.

The concentration samples were washed over a B.S. 200 sieve. The washed material retained on BS 200 mesh is dried in an oven at 50 °C. When dry, all the material is dry sieved through a set of sieves. As solids coarser than 1180 μm were all removed from the solid sample, they were made to pass through B.S. 50, 75, 100, 150, and 200 sieves (297, 210, 150, 106 and 75 μm sieves). Material retained on each sieve was weighed. Percentage retained on any sieve is calculated as

$$\frac{\text{Weight of solid particles retained}}{\text{Total weight of solid particles}} \times 100\%$$

This allowed complete determination of solids concentration profiles of six different sizes at different locations in the pipe cross-section at each velocity and efflux concentration.

At the end of the pipe loop a sampling point is provided in the vertical portions for collecting the delivered efflux sample. The efflux concentration of the slurry flowing through the pipeline was monitored by measuring the density of efflux sample. The efflux sample was stored in a bottle for determination of particle size distribution.

For measuring pressure drop, pressure taps along with separation chambers are provided at distance of 19.4 m in the pipeline. Separation chambers are provided at each pressure tap for interface separation of slurry and manometric fluid, water being the intermediate fluid. For better accuracy, pressure drop along the pipeline is measured by an inclined mercury manometer.

In the straight pipeline, a small length of perspex pipe was provided (designated as observation chamber) to establish the deposition velocity of the slurry in the pipeline by observing the motion of the particles at the bottom of the pipeline.

The static settled concentration is measured by allowing slurry of intermediate concentration to settle in a measuring jar till the level of the settled solids becomes static.

4. Experimental results

Measured vertical solids concentration profiles are shown in Figs. 1–3, where $C_{vj}(y')$ is the volumetric concentration of jth size particle at y' from bottom of pipeline, $y' = y/D$. Figs. 1–3

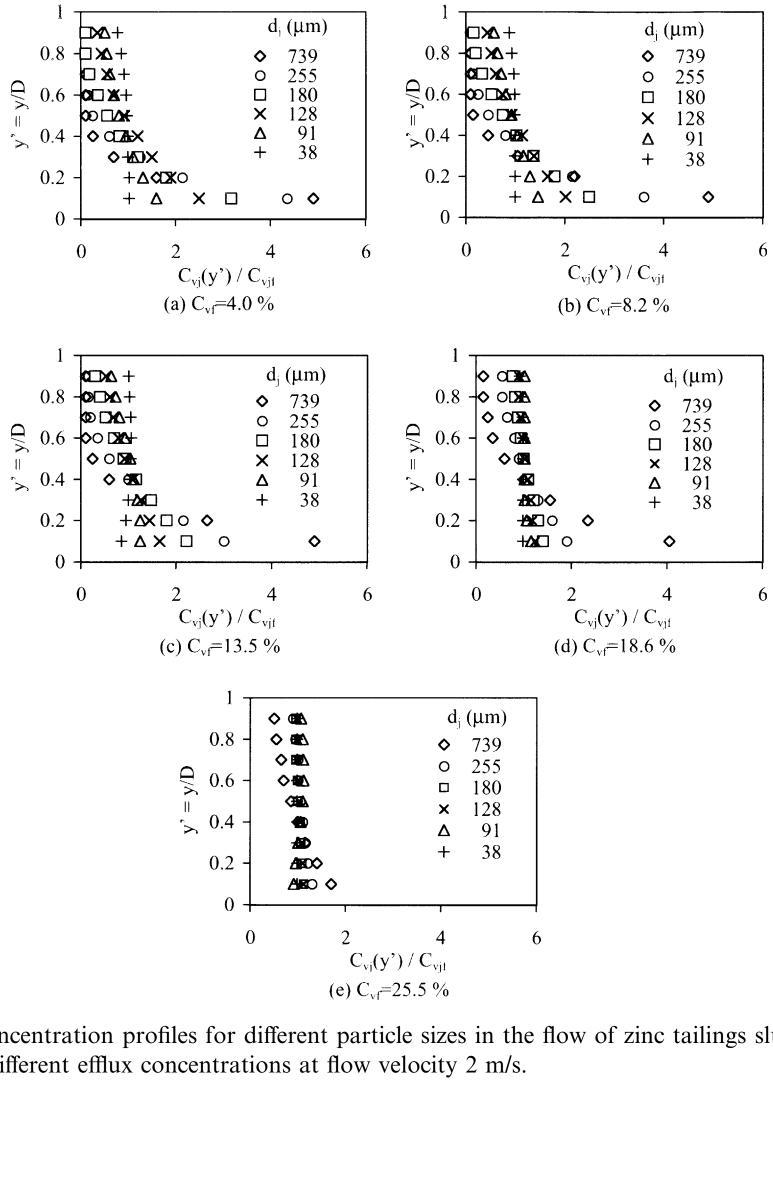

Fig. 1. Measured concentration profiles for different particle sizes in the flow of zinc tailings slurry through 105 mm diameter pipe with different efflux concentrations at flow velocity 2 m/s.

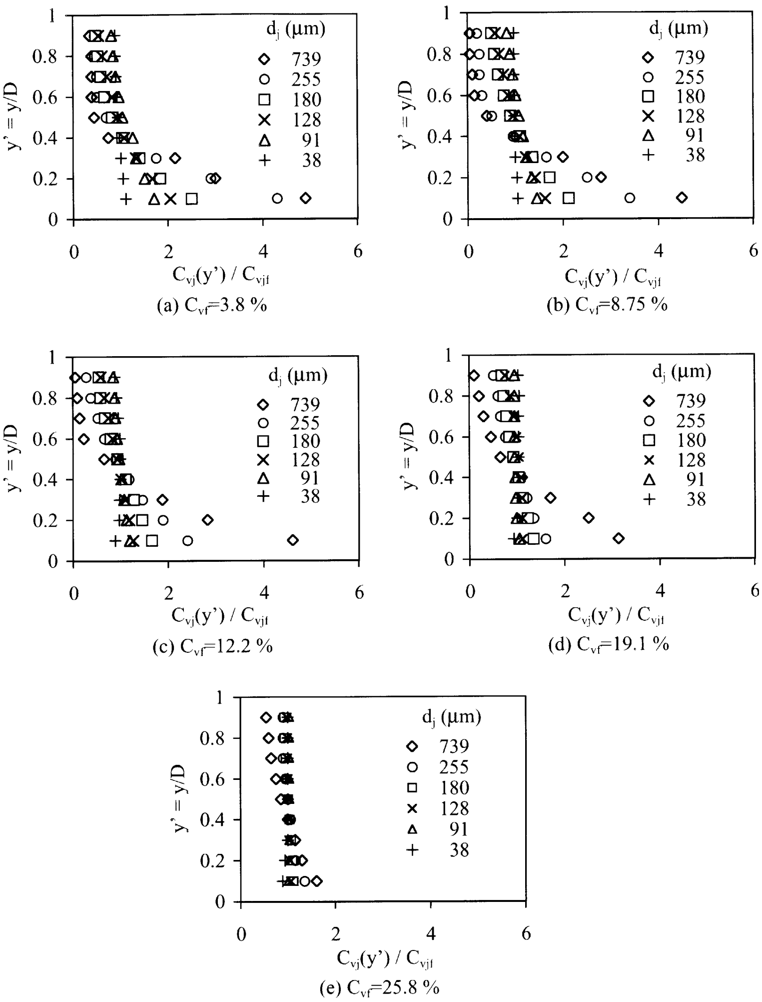

Fig. 2. Measured concentration profiles for different particle sizes in the flow of zinc tailings slurry through 105 mm diameter pipe with different efflux concentrations at flow velocity 2.75 m/s.

show the extent of redistribution of particles of six different sizes at three flow velocities of 2, 2.75 and 3.5 m/s using five efflux concentrations ranging from 4% to 26% by volume for each velocity. Generally we expect $C_{vj}(y')/C_{vif}$ to increase from top to bottom of the pipe. Further, for coarser size fractions, the ratio $C_{vj}(y')/C_{vif}$ can be expected to vary considerably. It is observed that the values of $(C_{vj}(y')/C_{vif}) = 1.0$ at all heights for 38 μm size particle at all concentrations and velocities tested, thereby indicating this particle size distributed homogeneously across the pipe cross-section. During experiments, it is observed that 38 μm size particle has homogeneous distribution across the pipe cross-section even at velocities close to the deposition velocity. The other size fractions are asymmetrically distributed with the degree of asymmetry increasing with increase in particle size. It is also observed that the degree of asymmetry in the solids concentration profiles for same concentration of slurry increases with decreasing velocity. This is expected

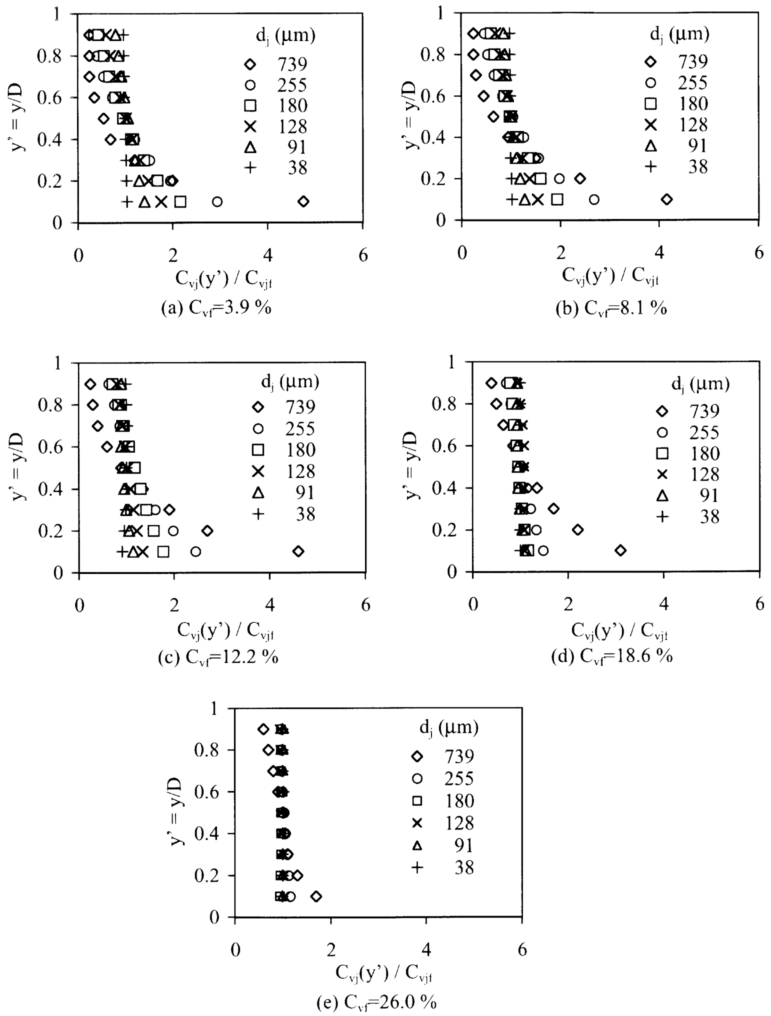

Fig. 3. Measured concentration profiles for different particle sizes in the flow of zinc tailings slurry through 105 mm diameter pipe with different efflux concentrations at flow velocity 3.5 m/s.

because with decrease in flow velocity there will be a decrease in turbulent energy which is responsible for keeping the solids in suspension. From these figures, it is also observed that for a given velocity, increasing concentration reduces the asymmetry in the vertical solids concentration profiles because of enhanced interference effect between solid particles. The effect of this interference is so strong that the asymmetry even at lower velocities is very much reduced at higher concentrations. Therefore it can be concluded that degree of asymmetry in the vertical solids concentration profiles depend upon the particle size, velocity of flow and concentration of slurry. From the observation of the shapes of the vertical concentration profile of each size (Figs. 1–3), the concentration profiles of coarser particles is highly skewed but as velocity increases the skewness in the concentration profiles of more particle sizes tend to reduce.

Some of the measured composite concentration profiles are shown in Fig. 9, where $C_v(y')$ is the composite volumetric concentration at y' from the pipe bottom. It is observed that the degree of asymmetry in the composite concentration profiles increases with decreasing velocity for same efflux concentration and for a particular flow velocity, it decreases with increasing efflux concentration. As discussed earlier, similar trends for the degree of asymmetry have been observed in the solids concentration profiles of different particle sizes.

Measurement of deposition velocity showed no significant change with efflux concentration. It is observed that deposition velocity increases only by a very little amount as efflux concentration increases. Deposition velocity varied from 1.1 to 1.18 m/s in the range of efflux concentration from 4% to 26% by volume. Similar observations for deposition velocity have been made by Schaan et al. (2000) in the flow of spherical glass beads, Ottawa sand and Lane mountain sand slurries through 105 mm diameter pipe. They found deposition velocity for spherical glass beads as constant over the range of solids concentrations from 5% to 45% by volume. For Ottawa sand and Lane mountain sand slurries, they reported only a little increase of around 0.2 m/s in deposition velocity in the range of solids concentrations from 5% to 30% by volume.

Pressure drops over 19.3 m of pipe length were measured for the multisized zinc tailings slurry flowing through 105 mm diameter pipe at five efflux concentrations ranging from 4% to 26% using eight flow velocities in the range from 1.2 m/s (slightly higher than the deposition velocity) to 4.0 m/s for each efflux concentration. These measured pressure drops per metre of pipe length are shown in Fig. 10. It is observed that at low velocities the slurry pressure drops are slightly higher than the corresponding values for water and with increase in velocity, pressure drop increases more rapidly. This can be attributed to the increased asymmetry in solids concentration profiles at lower velocities.

5. Comparison of prediction by existing models with experimental data

5.1. Comparison between measured and predicted solids concentration profiles based on Karabelas (1977) model

Figs. 4 and 5 show measured and predicted (by Karablas model) solids concentration profiles for efflux concentration of 4% by volume at flow velocity of 2 m/s and efflux concentration of 26.0% by volume at flow velocity of 3.5 m/s, respectively. It is observed that Karabelas model predicts concentration profile correctly for the finest particle size (38 μm). As the particle size increases, the Karabelas model predicts more asymmetric concentration profiles, thus overestimating the concentration at the bottom and underestimating the concentration at the top of pipe. For the largest particle size (739 μm) at lower velocities and efflux concentration, the degree of asymmetry increases by such an extent that Karabelas model predicts almost all the particles concentrated below $y' = 0.1$, thus underestimating the concentration for the entire range of y' from 0.1 to 0.9. As the flow velocity and efflux concentration increases the predicted concentration profiles for 739 μm particle size starts intersecting the measured profile in the range of y' from 0.1 to 0.9. For other particle sizes (91, 128, 180 and 255 μm) the Karabelas model predicts more asymmetric concentration overestimating the concentration at bottom and underestimating at the top in the range of y' from 0.1 to 0.9.

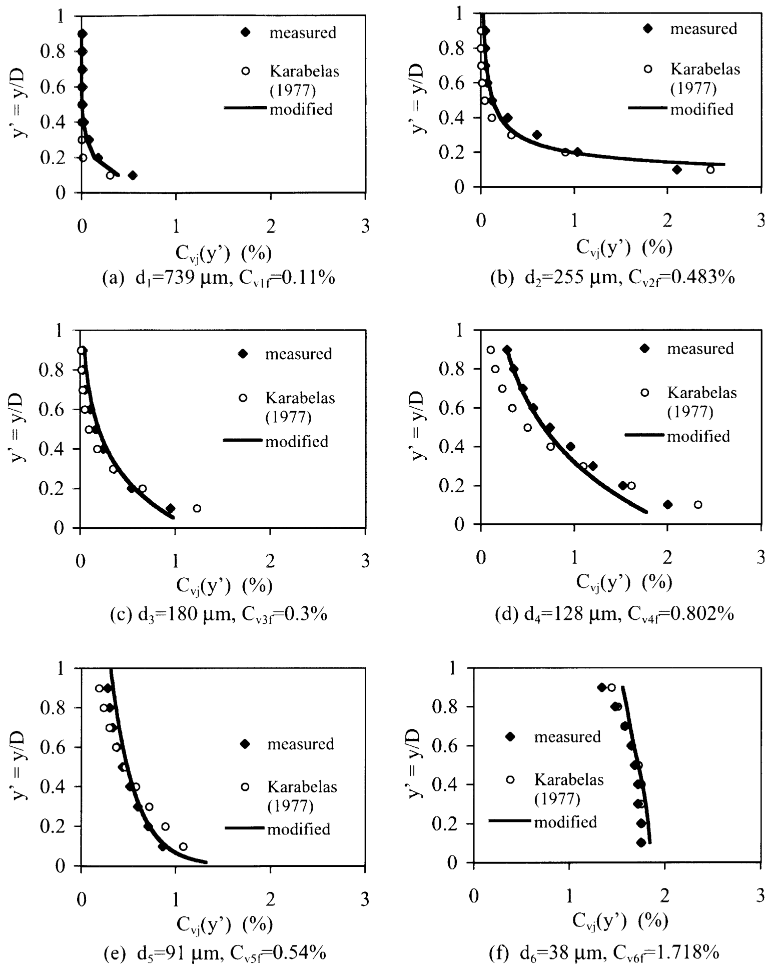

Fig. 4. Measured and predicted (by modified and Karabelas (1977) model) solids concentration profiles for zinc tailings slurry flowing through 105 mm diameter pipe at flow velocity of 2 m/s with efflux concentration of 4% by volume.

One of the parameters, which quantitatively represents the particle size distribution of the solids is the weighted mean diameter (d_{wmd}) defined as

$$d_{\mathrm{wmd}}(y') = \frac{\sum\{C_{\mathrm{v}j}(y')d_j\}}{\sum C_{\mathrm{v}j}(y')}, \quad j = 1, 2, \ldots\ldots, n \tag{27}$$

where d_j is the jth particle size.

Figs. 6–8 present the comparison between measured and predicted (by Karabelas model) overall weighted mean diameter. It is seen that for almost all the data except for a few data with lower efflux concentrations and velocities, the Karabelas model fails to predict accurately. The underestimation of concentration by Karabelas model for the largest size fraction results into the underestimation of weighted mean diameter across the entire range of y' from 0.1 to 0.9 at all the flow velocities and efflux concentrations except for very few data near pipe bottom.

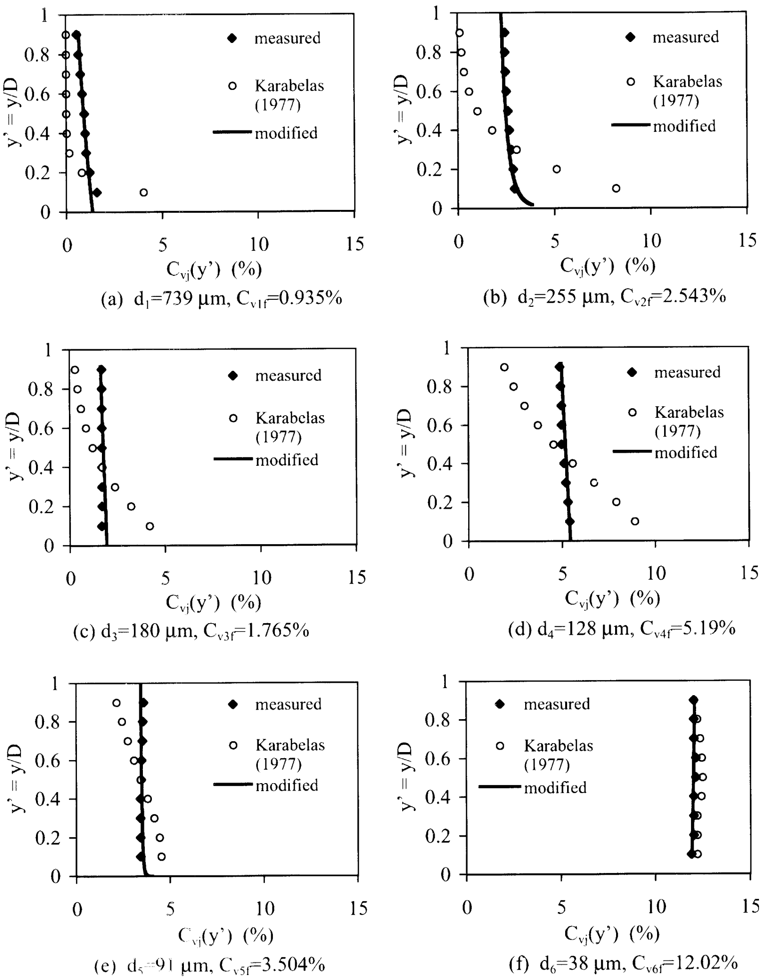

Fig. 5. Measured and predicted (by modified and Karabelas (1977) model) solids concentration profiles for zinc tailings slurry flowing through 105 mm diameter pipe at flow velocity of 3.5 m/s with efflux concentration of 26% by volume.

From the quantitative comparison, it is clear that Karabelas model is not taking into consideration the changes in fluid and flow properties, which occur with increase in efflux concentration and particle diameter. The causes for failure of Karabelas model at higher efflux concentrations and coarser particles are identified as

(i) In his final solution given by Eq. (3), he used unhindered settling velocity in the calculations not accounting for the effect of concentration, particle size distribution and pipe walls. Richardson and Zaki (1954) have already given following correlation for hindered settling velocity by taking into consideration the above factors:

$$w_j = w_{j0}(1 - C_{vf})^Z \tag{28}$$

where for

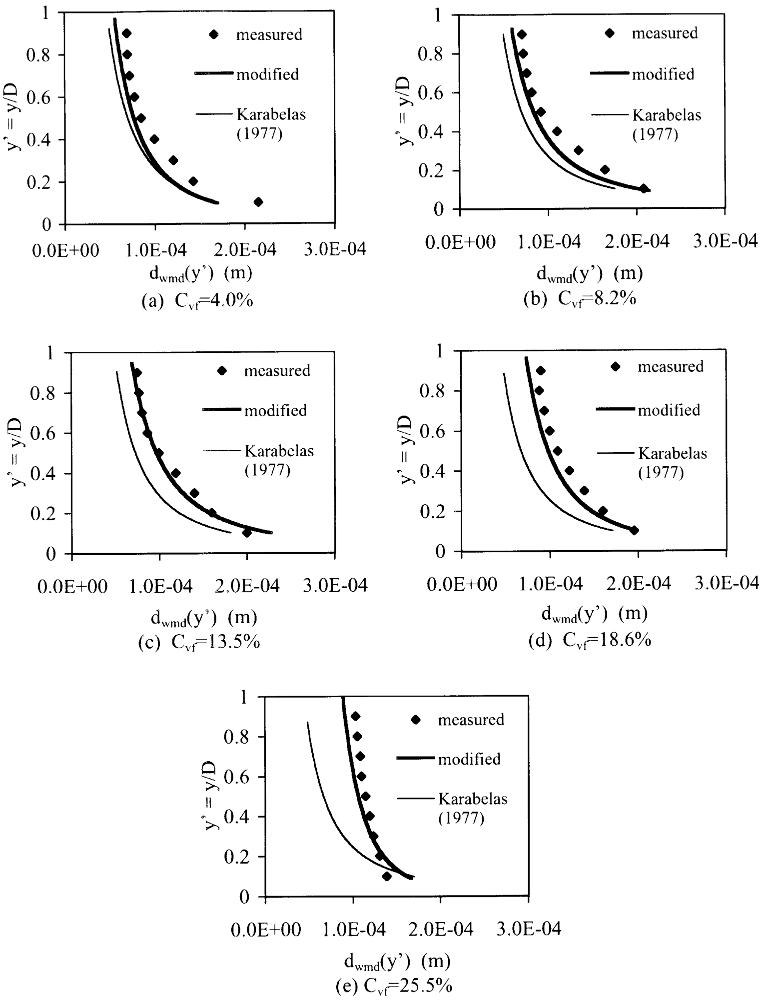

Fig. 6. Measured and predicted (by modified and Karabelas (1977) model) overall weighted mean diameter profiles for zinc tailings slurry flowing through 105 mm diameter pipe with different efflux concentrations at flow velocity 2 m/s.

$$0.002 < Re_{\mathrm{d}} \leqslant 0.2; \quad Z = 4.65 + 1.95\left(\frac{d_j}{D}\right)$$

$$0.2 < Re_{\mathrm{d}} \leqslant 1.0; \quad Z = \left[4.35 + 17.5\left(\frac{d_j}{D}\right)\right] Re_{\mathrm{d}}^{-0.03}$$

$$1.0 < Re_{\mathrm{d}}; \quad Z = \left[4.45 + 18.0\left(\frac{d_j}{D}\right)\right] Re_{\mathrm{d}}^{-0.1}$$

(ii) Particle diffusivity ε_{s} is assumed as constant and equal to liquid diffusivity ε_{l}. Wasp et al. (1977), Raudkivi (1990), Govier and Aziz (1982), Walton (1995) and Kaushal et al. (2002) have suggested following expression for particle diffusivity:

$$\varepsilon_{\mathrm{s}} = \beta\varepsilon_{\mathrm{l}} \tag{29}$$

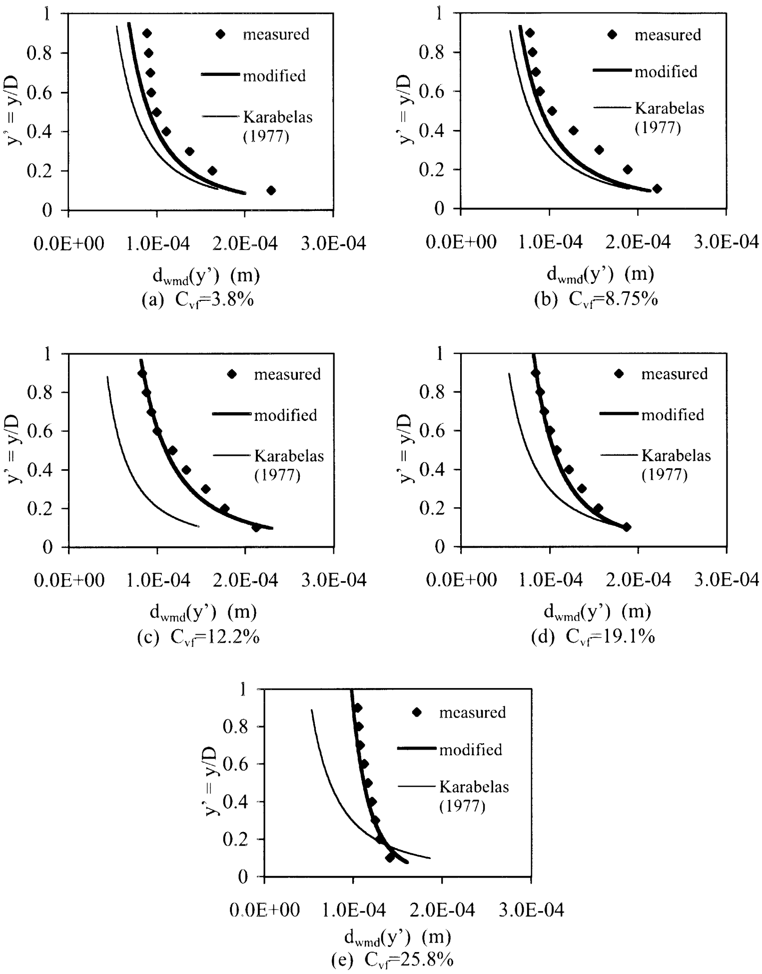

Fig. 7. Measured and predicted (by modified and Karabelas (1977) model) overall weighted mean diameter profiles for zinc tailings slurry flowing through 105 mm diameter pipe with different efflux concentrations at flow velocity 2.75 m/s.

where β is the dimensionless particle diffusivity (or dimensionless particle diffusion coefficient) and ε_l is the liquid diffusivity (or momentum transfer coefficient).

Further, Karabelas also assumed that ε_l is constant across the cross-section of the pipe. The assumption of liquid diffusivity being constant is not valid. The liquid diffusivity is not constant across the pipe cross-section due to turbulent motion and its variation depends on several parameters like pipe diameter, flow conditions, etc. As suggested by Longwell (1977) the liquid diffusivity in turbulent pipe flow is given by:

$$\varepsilon_l = 0.369Ru*\frac{y}{R}\left(1-\frac{y}{R}\right) \quad \text{for } 0 \leqslant y/D \leqslant 0.337$$

$$\varepsilon_l = 0.0775Ru* \quad \text{for } 0.337 \leqslant y/D \leqslant 0.663$$

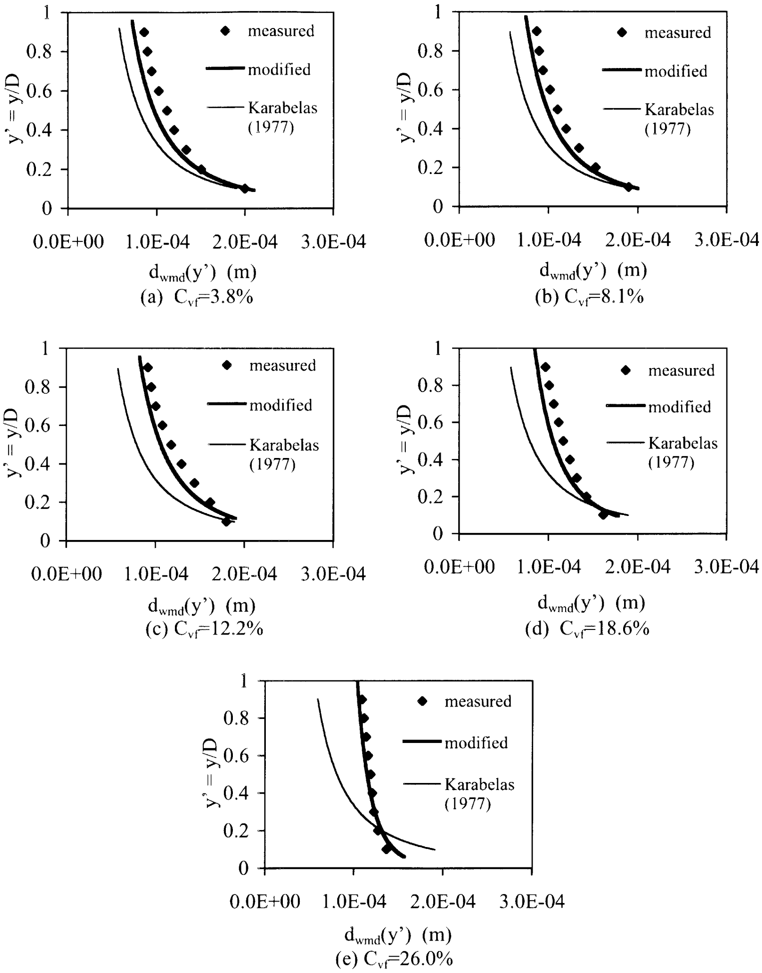

Fig. 8. Measured and predicted (by modified and Karabelas (1977) model) overall weighted mean diameter profiles for zinc tailings slurry flowing through 105 mm diameter pipe with different efflux concentrations at flow velocity 3.5 m/s.

$$\varepsilon_l = 0.369Ru * \left(\frac{y}{R} - 1\right)\left(2 - \frac{y}{R}\right) \quad \text{for } 0.663 \leqslant y/D \leqslant 1.0 \tag{30}$$

On the basis of extensive analysis of composite concentration profiles, Kaushal et al. (2002) have already given following correlation for dimensionless particle diffusivity β by taking into consideration the effect of solid concentration and static settling concentration:

$$\beta = 1.0 + 0.12504e^{4.22054C_{vf}/C_{vss}} \tag{31}$$

5.2. Comparison between measured and predicted pressure drops based on Wasp et al. (1977) and Gillies et al. (1991) models

Fig. 10(a)–(e) show the measured and predicted pressure drops for the multisized zinc tailings slurry flowing through 105 mm diameter pipe at five efflux concentrations ranging from 4% to

26% using eight flow velocities in the range from 1.2 to 4.0 m/s for each efflux concentration. From these figures, it is found that the Wasp model predicts pressure gradient with reasonable accuracy at lower efflux concentrations at all the flow velocities and at smaller flow velocities near deposition velocity for all efflux concentrations considered in the present study. However, Wasp model overestimates the pressure drops for higher efflux concentrations at larger flow velocities. Comparison of pressure drop data with the predictions by two layer model of Gillies et al. (1991) is satisfactory.

From the comparison, it is clear that Wasp model is not taking into consideration the changes in fluid and flow properties which occurs with increase in efflux concentration and flow velocity. The causes for failure of Wasp model at higher efflux concentrations and flow velocities are identified as:

(a) It has been established that the dimensionless particle diffusivity is not equal to unity as considered by Wasp. Kaushal et al. (2002) have given a correlation for particle diffusivity by taking into consideration the effect of solid concentration.
(b) At higher flow velocities, the vehicle pressure drop has a major share in total pressure drop. Mukhtar (1991) has proposed following modified Wood's equation for vehicle friction factor by taking into consideration the effect of concentration for multisized particulate slurry transportation through pipe:

$$f_m = (a + bRe_m^{-c})(1 - 0.33C_{\mathrm{wf}}) \tag{32}$$

where C_{wf} is the efflux concentration by weight.

Hence an attempt has been made in the present study to modify the Karabelas and Wasp et al. models by alleviating some of the restrictive assumptions used in the models.

6. Description of modified models

6.1. Modified Karabelas model

General solution given by Eq. (1) is used instead of approximately closed form solution given by Eq. (3). A computer program for Karabelas general solution with modifications has been developed. The modifications incorporated are given below:

1. Hindered settling velocity w_j for each particle size is calculated using the Eq. (28) given by Richardson and Zaki (1954). In the current modifications, the provision for allowing any variation of w_j across the pipe cross-section has also been incorporated. It is to be noted that w_j is a function of local solid concentration which is not known a priori. Hence, in the first iteration, w_j corresponding to efflux concentration is used to compute the overall concentration profile. In the subsequent iterations, the computed values of local concentrations are used to calculate $w_j(y)$ at each point. This procedure is repeated until convergence is obtained.
2. Eq. (31) proposed by Kaushal et al. (2002) has been used to calculate the local values of β in the modified model considering C_{vf} as the local concentration. It is to be noted that in Eq. (1) both

w_j and ε_s are complicated function of local solid concentration and hence the calculation procedure has to be iterative.

6.2. Modified wasp model

Following modifications have been made in the existing Wasp model :

1. In the first iteration f_m, the Fanning friction factor has been evaluated by Eq. (32), the modified Wood's equation proposed by Mukhtar (1991).
2. In the second iteration, for each particle size, the ratio of solids in vehicle and bed is calculated by Eq. (12) proposed by Wasp et al. (1977) using the value of dimensionless particle diffusivity as proposed by Kaushal et al. (2002) in Eq. (31).

7. Comparison of predictions by modified models with measured values

7.1. Solids concentration profiles

Figs. 4 and 5, Figs. 6–8 and Fig. 9 present the measured and predicted (by modified Karabelas model) solids concentration profiles, overall weighted mean diameter profiles and composite

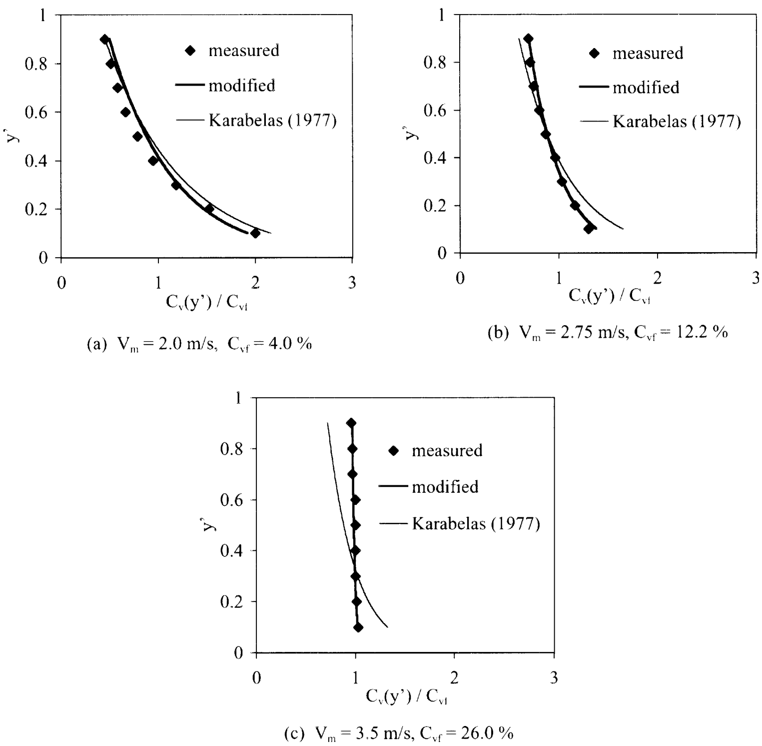

Fig. 9. Some measured and predicted (by modified and Karabelas (1977) model) composite concentration profiles for zinc tailings slurry flowing through 105 mm diameter pipe.

concentration profiles, respectively. Comparison of modified model with experimental data show good agreements, except for a few data at lower efflux concentrations.

7.2. Pressure drops

Experimental and predicted pressure drops by modified Wasp model for the multisized zinc tailings slurry flowing through 105 mm diameter pipe at five efflux concentrations ranging from 4% to 26% using eight flow velocities in the range from 1.2 to 4.0 m/s for each efflux concentration

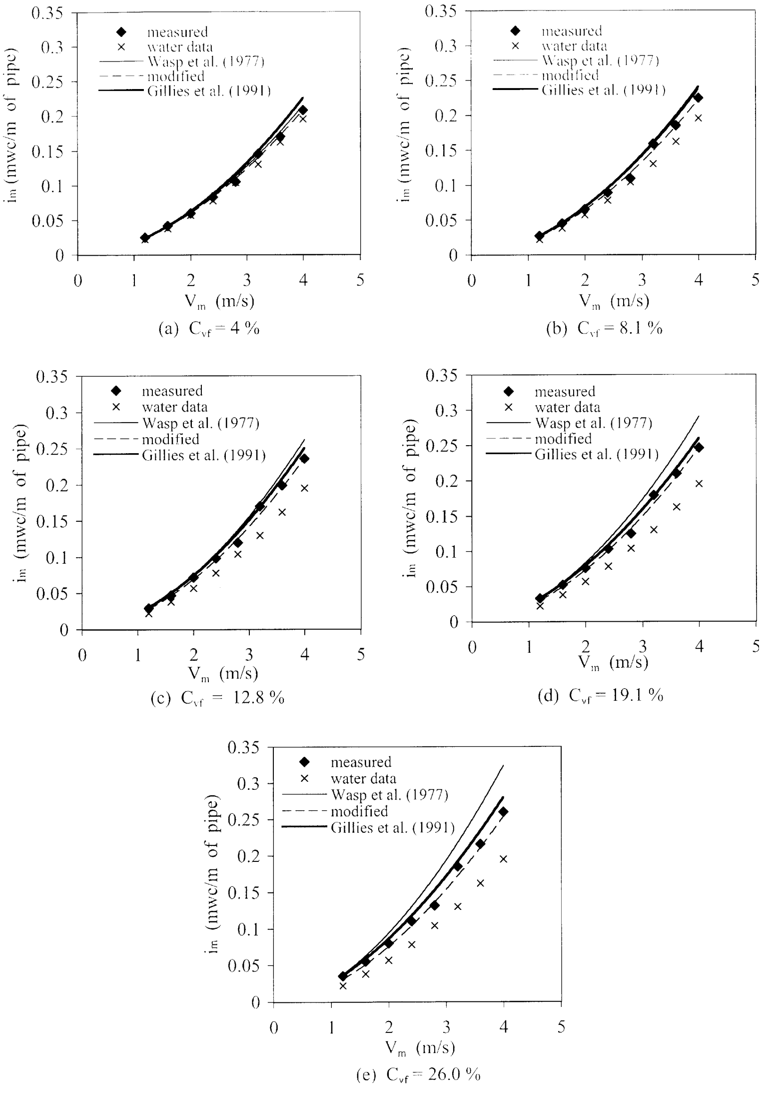

Fig. 10. Comparison between measured and predicted (by Gillies et al. (1991), Wasp et al. (1977) and modified model) pressure drops for zinc tailings slurry with different efflux concentrations flowing through 105 mm diameter pipe.

have been plotted in Fig. 10 (a)–(e). Comparison of modified model with experimental data shows good agreement.

8. Conclusions

Following conclusions have been drawn on the basis of present study:

(a) Vertical concentration profiles for six particle sizes ranging from 38 to 739 μm were measured for multisized particulate zinc tailings slurry flowing through 105 mm diameter pipe. Experiments were conducted at three flow velocities of 2, 2.75 and 3.5 m/s using five efflux concentrations ranging from 4% to 26% by volume for each velocity.
(b) Karabelas (1977) model for prediction of solids concentration profiles has been modified by alleviating some of the restrictive assumptions. Comparison of modified model with experimental data shows good agreement.
(c) Experimental data for pressure drop were collected for multisized zinc tailings slurry flowing through 105 mm diameter pipe at five efflux concentrations ranging from 4% to 26% using eight flow velocities in the range from 1.2 to 4.0 m/s for each efflux concentration.
(d) Wasp et al. (1977) model for pressure drop prediction has been modified by alleviating some of the restrictive assumptions. Comparison of modified model with experimental data shows good agreement. Comparison of pressure drop data with the predictions by two layer model of Gillies et al. (1991) is satisfactory.
(e) It is recommended for further study to find out the dependence of particle diffusion coefficient (β) on particle size, particle shape and other fluid–particle relationships besides efflux concentration of slurry.

References

Doron, P., Granica, D., Barnea, D., 1987. Slurry flow in horizontal pipes—experimental and modeling. Int. J. of Multiphase Flow 13, 535–547.

Ghanta, K.C., Purohit, N.K., 1999. Pressure drop prediction in hydraulic transport of bi-dispersed particles of coal and copper ore in pipeline. The Canadian Journal of Chemical Engineering 77, 127–131.

Gillies, R.G., Shook, C.A., 2000. Modelling high concentration settling slurry flows. The Canadian Journal of Chemical Engineering 78, 709–716.

Gillies, R.G., Hill, K.B., Mckibben, M.J., Shook, C.A., 1999. Solids transport by laminar Newtonian flows. Powder Technology 104, 269–277.

Gillies, R.G., Shook, C.A., Wilson, K.C., 1991. An improved two layer model for horizontal slurry pipeline flow. The Canadian Journal of Chemical Engineering 69, 173–178.

Govier, G.W., Aziz, K., 1982. The Flow of Complex Mixtures in Pipes. Krieger Publication, Malabar, FL.

Hunt, J.N., 1954. The turbulent transport of suspended sediment in open channels. Royal Society of London, Proc., Series A 224 (1158), 322–335.

Ismail, H.M., 1952. Turbulent transfer mechanism and suspended sediment in closed channels. Transactions of ASCE 117, 409–446.

Karabelas, A.J., 1977. Vertical distribution of dilute suspensions in turbulent pipe flow. AIChE Journal 23, 426–434.

Kaushal, D.R, Tomita, Y., Dighade, R.R., 2002. Concentration at the pipe bottom at deposition velocity for transportation of commercial slurries through pipeline. Powder Technology 125, 89–101.

Longwell, P.A., 1977. Mechanics of Fluid Flow. McGraw-Hill Book Company, New York, USA.
Mishra, R., Singh, S.N., Seshadri, V., 1998. Improved model for prediction of pressure drop and velocity field in multi-sized particulate slurry flow through horizontal pipes. Powder Handling and Processing Journal 10, 279–289.
Mukhtar, A., 1991. Investigations of the flow of multisized heterogeneous slurries in straight pipe and pipe bends. Ph.D. Thesis, IIT, Delhi.
O'Brien, M.P., 1933. Review of the theory of turbulent flow and its relations to sediment transport. Transaction of the American Geophysical Union 14, 487–491.
Raudkivi, A.J., 1990. Loose Boundary Hydraulics. Pergamon Press, Oxford, England.
Richardson, J.F., Zaki, W.M., 1954. Sedimentation and fluidization: part-1. Transactions of the Institution of Chemical Engineers 32, 35–53.
Roco, M.C., Shook, C.A., 1983. Modelling of slurry flow, the effect of particle size. The Canadian Journal of Chemical Engineering 61, 494–503.
Roco, M.C., Shook, C.A., 1984. Computational methods for coal slurry pipeline with heterogeneous size distribution. Powder Technology 39, 159–176.
Rouse, H., 1937. Modern conceptions of the mechanics of fluid turbulence. Transactions of ASCE 102, 463–505.
Schaan, J., Sumner, R.J., Gillies, R.G., Shook, C.A., 2000. The effect of particle shape on pipeline friction for Newtonian slurries of fine particles. The Canadian Journal of Chemical Engineering 78, 717–725.
Seshadri, V., Malhotra, R.C., Sundar, K.S., 1982. Concentration and size distribution of solids in a slurry pipeline. Proc., 11th National Conference on Fluid Mechanics and Fluid Power, BHEL, Hyderabad, India, pp. 110–123.
Shook, C.A., Daniel, S.M., 1965. Flow of suspensions of solids in pipeline: I. Flow with a stable stationary deposit. The Canadian Journal of Chemical Engineering 43, 56–72.
Shook, C.A., Daniel, S.M., Scott, J.A., Holgate, J.P., 1968. Flow of suspensions in pipelines. The Canadian Journal of Chemical Engineering 46, 238–244.
Sundqvist, A., Sellgren, A., Addie, G., 1996. Slurry pipeline friction losses for coarse and high density products. Powder Technology 89, 19–28.
Thomas, D.G., 1965. Transport characteristics of suspensions: VIII. A note on the viscosity of Newtonian suspensions of uniform spherical particles. Journal of Colloidal Science 20, 267–277.
Walton, I.C., 1995. Eddy diffusivity of solid particles in a turbulent liquid flow in a horizontal pipe. AIChE Journal 41, 1815–1820.
Wasp, E.J., 1963. Cross country coal pipeline hydraulics. Pipeline News 35, 20–25.
Wasp, E.J., Aude, T.C., Kenny, J.P., Seiter, R.H., Jacques, R.B., 1970. Deposition velocities, transition velocities and spatial distribution of solids in slurry pipelines. Proc. Hydrotransport 1, BHRA Fluid Engineering, Coventry, UK, paper H4.2, pp. 53–76.
Wasp, E.J., Kenny, J.P., Gandhi, R.L., 1977. Solid Liquid Flow Slurry Pipeline Transportation, first ed. Trans. Tech. Publications, Clausthal, Germany.
Wilson, K.C., 1988. Evaluation of interfacial friction for pipeline transport models. Proceedings of the 11th International Conference on Hydraulic Transport of Solids, BHRA Fluid Engineering, Cranfield, UK, pp. 107–116.
Wilson, K.C., 1976. A unified physically-based analysis of solid–liquid pipeline flow. Proceedings of the 4th International Conference on Hydraulic Transport of Solids, BHRA Fluid Engineering, Cranfield, UK, Paper A2, pp. 1–16.
Wilson, K.C., Clift, R., Sellgren, A., 2002. Operating points for pipelines carrying concentrated heterogeneous slurries. Powder Technology 123, 19–24.